APPROXIMATE SOLUTION OF THE BOLTZMANN EQUATION FOR ELECTRON DISTRIBUTION FUNCTIONS IN A WEAKLY IONIZED MOLECULAR PLASMA IN A CONSTANT ELECTRICAL FIELD

G. Ya. Dynnikova

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Computation of the electron distribution function f(U) turns out to be necessary in solving a number of problems associated with determining the efficiency of processes in a plasma. The electron distribution function is described by an integro-differential equation whose solution is of known difficulty. A number of numerical solution methods [1-4] as well as an analytical investigation of the electron distribution function [5-8] exist. However, the search for simpler methods of solution permitting the fundamental electron energetic characteristics to be found with minimal time expenditure and satisfactory accuracy continues to be an urgent problem.

An approximate analytic expression is found in this paper for the electron distribution function possessing high simplicity and clearness and permitting calculation of f(U) upon satisfaction of definite conditions with high accuracy. In those cases when the accuracy turns out to be inadequate, this expression can be used as the initial approximation in an iteration process. The solution is found by taking into account inelastic collisions of the first kind. Moreover, an expression is obtained for the electron distribution function by taking account of collisions of the second kind which, although it is indeeed not explicit, permits easy evaluation of f(U).

In the quasistationary case of a spatially homogeneous gas and an almost isotropic electron velocity distribution function as well as a small degree of ionization, the kinetic equation for the isotropic part of the electron velocity distribution function (expressed as a function of the energy) with inelastic collisions taken into account has the form [2]

$$\frac{1}{3}\frac{E^2Ue^2}{NQ_{ts}}\frac{df}{dU}+\frac{2m}{M}U^2NQ_{ts}\left(f+kT\frac{df}{dU}\right)+N\varepsilon_{rc}UQ_{rc}\left(f+kT\frac{df}{dU}\right)+\sum_i\int_U^{U+U_i}NUQ_i(U)f(U)\,dU, \qquad (1)$$

where E is the electrical field intensity, U, e, m are the electron energy, charge, and mass, M is the molecule mass, k is the Boltzmann constant, N is the molecule density, T is the gas temperature, U_i is the energy lost by an electron in inelastic collisions with molecules, Q_{ts} is the transport section of electron scattering by a molecule, Q_i is the inelastic interaction section, ε_{rc} is a rotational constant, Q_{rc} is defined by a formula analogous to [9, 10]

$$Q_{rc}=\sum_i\sum_n\frac{N_i}{N}\frac{U_{i,i+n}^2}{\varepsilon_{rc}kT}Q_{i,i+n}.$$

Here N_i is the molecule density at the i-th rotational level, $U_{i,i+n}$, $Q_{i,i+n}$ are the energy and section of the transition from the i-th rotational level to the (i + n)-th.

The first component describes the electron energy increase in an electrical field, the second is the energy loss in elastic collisions, the third is the loss by excitation of molecule rotational levels in a diffusion approximation [1, 10] and under the assumption of an equilibrium molecule distribution over rotational levels, the fourth is energy transmission to the vibrational and electron degrees of freedom, as well as the energy loss by ionization with collisions of the first kind taken into account.

Let us introduce the notation $A=\frac{1}{3}\frac{E^2Ue^2}{NQ_{ts}}+\frac{2m}{M}U^2NQ_{ts}kT+\varepsilon_{rc}UNQ_{rc}kT$, $B=\frac{2m}{M}U^2NQ_{ts}+N\varepsilon_{rc}UQ_{rc}$, $C_i=NUQ_i$, which is used to give (1) the form

$$A\frac{df}{dU}+Bf+\sum\int_U^{U+U_i}C_i(U')f(U')\,dU'=0. \qquad (2)$$

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We shall seek the solution f(U) in the form

$$f(U) = f_0 \exp\left[-\int_0^U Y(U')\,dU'\right],$$

where $df/dU = -Y(U)f(U)$. Let us note that the function Y(U) is always positive, as is seen from (2), since A, B, and C_i are positive. We use this later.

Let us show that if $Y(U') \gg 1/U_i$ in the domain $U \leq U' \leq U + \varepsilon$, where ε satisfies the inequality $1/Y(U) \ll \varepsilon < U_i$ and the relative change in the functions Y(U) and $C_i(U)$ is small, then the approximation

$$\int_U^{U+U_i} C_i(U')\,f(U')\,dU' \approx C_i(U)\,f(U)/Y(U) \tag{3}$$

is valid. In fact, if Y(U) is large, then f(U) is a fast dropping function and the main contribution to the integral will be given by the domain of the dimension ε. Therefore

$$\int_U^{U+U_i} C_i(U')\,f(U')\,dU' \approx f(U)\int_0^{\varepsilon} C_i(U+U')\exp\left[-\int_0^{U'} Y(U'')\,dU''\right]dU' \approx$$

$$\approx f(U)\,C_i(U)\int_0^{\varepsilon} e^{-Y(U)U'}\,dU' = \frac{f(U)\,C(U)}{Y(U)}\left(1 - e^{-Y(U)\varepsilon}\right) \approx f(U)\,C(U)/Y(U).$$

The approximation (3) permits conversion of the integro-differential equation (2) into an algebraic equation in Y(U): $AY^2 - BY - C = 0$, $C = \Sigma C_i$, which has two roots, positive and negative. As mentioned above, $Y(U) > 0$; consequently

$$Y(U) = \frac{1}{2}\left(\frac{B}{A} + \sqrt{\frac{B^2}{A^2} + \frac{4C}{A}}\right) \tag{4}$$

(Y(U) is the logarithmic derivative of the function f(U), i.e., equals the tangent of the slope of the graph of f(U) constructed in a logarithmic scale).

Let us examine the physical meaning of the quantities in the expression for Y(U). We introduce the notation $U_{ela} = 2mU/M$, $U_{rc} = Q_{rc}\varepsilon_{rc}/Q_{ts}$, $U_e = E_e(NQ_{ts})$, $U_T = kT$. Then

$$\frac{B}{A} = \frac{U_{ela} + U_{rc}}{\frac{1}{3}U_e^2 + U_{ela}U_T + U_{rc}U_T}, \quad \frac{C}{A} = \frac{1}{\frac{1}{3}U_e^2 + U_{ela}U_T + U_{rc}U_T}\sum_i \frac{Q_i}{Q_{ts}},$$

where U_e is the characteristic magnitude of the electron energy increment in an electrical field between two collisions with molecules, U_{ela}, U_{rc} are characteristic magnitudes of the energy lost by an electron in an elastic collision and in a collision with excitation of rotation, and U_T is the thermal energy of the gas molecules.

The solution (4) is obtained under the assumption

$$Y(U) \gg 1/U_i. \tag{5}$$

It is seen from (4) that $Y(U) \geq B/A$, $Y(U) \geq \sqrt{C/A}$ and at the same time $Y(U) \leq B/A + \sqrt{C/A}$. This means that it is necessary to satisfy condition (5) so that at least one of the quantities

$$\alpha = \frac{BU_{min}}{A} = \frac{(U_{ela} + U_{rc})\,U_{min}}{\frac{1}{3}U_e^2 + U_{ela}U_T + U_{rc}U_T}, \quad U_{min} = \min_i\{U_i\},$$

$$\beta = \sqrt{\frac{CU_{min}^2}{A}} = \sqrt{\frac{\sum Q_i}{Q_{ts}}\,\frac{U_{min}^2}{\frac{1}{3}U_e^2 + U_{ela}U_T + U_{rc}U_T}}$$

would be much less than unity.

Presented in Fig. 1 are graphs of the functions α and β for a cold plasma (T = 300 K) of pure carbon dioxide for values of the E/N ratio characteristic for electron-ionization CO lasers: 1) $0.707\cdot10^{-16}$ and 2) $0.5\cdot10^{-16}$ V·cm^2. Represented in Fig. 2 are the dependences $Q_i(U)$

from [11] modified according to [12] and used in the computations. The effective excitation sections of the rotational degrees of freedom are taken exactly the same as in [12].

It is seen from Fig. 1 that for given values of the parameters in the domain where the influence of the inelastic collisions is substantial, the inequality (5) is satisfied. In the prethreshold domain ($U \leq 0.3$ eV) a noticeable difference in the values of Y(U) calculated by means of (4) from the exact value can be expected since the approximation (3) is not valid in this domain.

Displayed in Fig. 3 are the dependences Y(U) obtained by means of (4) (solid lines) and by a numerical solution of (1) (dashed lines) for $E/N = 0.707 \cdot 10^{-16}$, $0.5 \cdot 10^{-16}$, $0.354 \cdot 10^{-16}$ V·cm^2 (curves 1-3). It is seen that good agreement holds everywhere except in the prethreshold domain. However, the extent of this domain is relatively small and since the integral of Y(U) is in the expression for the distribution function, it can then be expected that its influence on f(U) will be negligible.

Shown in Fig. 4 are graphs of the function f(U) (the notation is the same as in Fig. 3). It is seen that despite the sufficiently rough approximation (3), good agreement holds between the results. The accuracy of the solution in the prethreshold domain can be raised if an approximation of the integral that uses the smallness of the quantity $U_i Y(U)$

$$\int_U^{U+U_i} C_i f dU \approx f(U)\, e^{-(U_i-U)Y(U)} \int_U^{U+U_i} C_i\, dU \approx f(U)\,[1-(U_i-U)\,Y] \int_U^{U+U_i} C_i dU$$

is taken instead of the approximation (3). Then

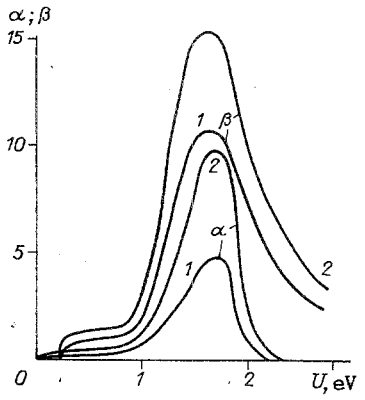

Fig. 1

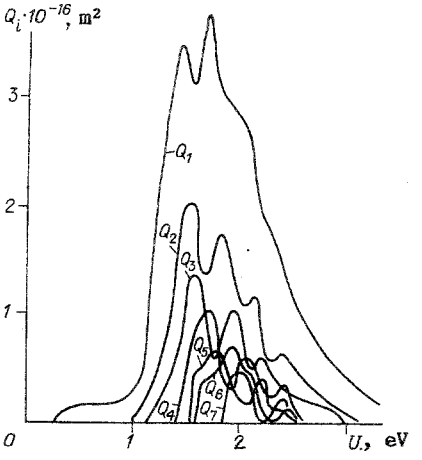

Fig. 2

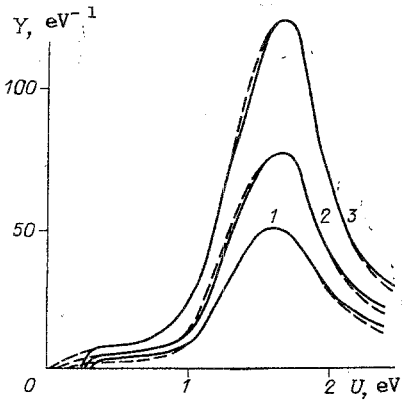

Fig. 3

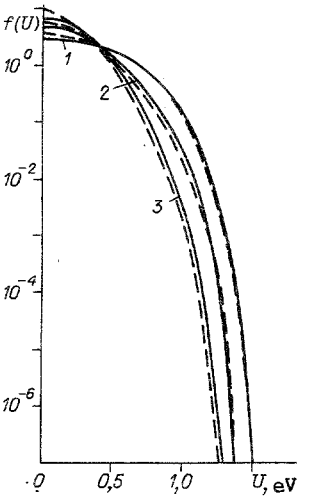

Fig. 4

$$Y(U)=\frac{B+\sum\limits_{U}^{U+U_i} C_i\,dU}{A+\sum(U_i-U)\int\limits_{U}^{U+U_i} C_i\,dU}.$$

The summation must here be performed only over those i for which $YU_i \leq 1$, which requires additional analysis. In practice it usually turns out to be sufficient to take one term of the sum corresponding to the minimal value of U_i.

As already mentioned above, when higher accuracy is required, the approximate solution obtained in this paper can be used as the initial approximation $f_1(U)$ in the iteration process [3], and the next approximation will be $Y(U)=\frac{1}{A}\left(B+\frac{1}{f_1(U)}\sum\int\limits_{U}^{U+U_i} C_i f_1(U)\,dU\right)$.

The method described above can be extended to the case when the influence of collisions of the second kind must be taken into account. The equation for f(U) here has the form

$$A\frac{df}{dU}+Bf+\sum_i\int\limits_{U}^{U+U_i} C_i f\,dU-\sum_j\int\limits_{U-U_j}^{U} Q_{-j}UN_j f\,dU=0. \tag{6}$$

Going over to the variable Y and approximating the integral in the third component by means of (3), we obtain

$$Y(U)=\frac{1}{2}\left(\frac{B'}{A}+\sqrt{\left(\frac{B'}{A}\right)^2+\frac{4C}{A}}\right), \tag{7}$$

where $B'=B-\frac{1}{f(U)}\sum_j\int\limits_{U-U_j}^{U} Q_{-j}N_jU'f(U')\,dU'$. In contrast to (4) the expression (7) is not explicit since Y(U) must be known for $U < U_0$ to evaluate Y(U) at the point $U = U_0$. However, finding Y(U) and f(U) numerically in this case is much simpler than the solution for the integro-differential equation (6).

Therefore, it is shown that the approximate method of computing the electron energy distribution function is simple and efficient. It affords the possibility of a simple analysis of the influence of different processes and some of their parameters on the distribution function and, respectively, on the macroscopic characteristics of a weakly ionized plasma and permits finding the electron distribution function with good accuracy for sufficiently small values of E/N and the plasma temperature that is realized in many practical problems.

LITERATURE CITED

1. B. Sherman, "The difference-differential equation of electron energy distribution in a gas," J. Math. Anal. Appl., 1, No. 3 (1960).
2. L. S. Frost and A. V. Phelps, "Rotation and momentum transfer cross section for electrons in H_2 and N_2 from transport coefficients," Phys. Rev., 127, No. 5 (1962).
3. J. Lucas, "Energy distributions for electron swarms in hydrogen at high values of E/p," Int. J. Electron., 27, No. 3 (1969).
4. C. Smith and R. Thomson, Numerical Modeling of Gas Lasers [Russian translation], Mir, Moscow (1981).
5. L. M. Kovrizhnykh, "Influence of inelastic collisions on the electron velocity distribution," Zh. Éksp. Teor. Fiz., 37, No. 2(8) (1959).
6. Yu. M. Kagan and R. I. Lyagushchenko, "On electron energy distribution functions in a positive discharge column," Zh. Tekh. Fiz., 34, No. 5, (1964).
7. A. I. Lukovnikov, E. P. Fetisov, and E. S. Terekhov, "On free electron distribution functions in a low-temperature plasma," Zh. Tekh. Fiz., 40, No. 9 (1970).
8. L. D. Tsendin, "On the influence of inelastic impacts on the electron distribution function in an electrical field," Zh. Tekh. Fiz., 41, No. 11 (1971).
9. B. Ya. Moizhes and G. E. Pikus (eds.), Thermoemission Converters and Low-Temperature Plasma [in Russian], Nauka, Moscow (1973).

10. F. G. Baksht and V. G. Ivanov, "Electron distribution function in a near-electric layer of a molecular gas plasma," Zh. Tekh. Fiz., 5, No. 10 (1984).
11. G. I. Shulz, "Vibrational excitation of N_2, CO_2 and H_2 by electron impact," Phys. Rev., 135A, No. 4 (1964).
12. M. G. Basov, V. I. Dolinina, I. B. Kovsh, et al., Self-consistent Analysis of the Kinetics of Elementary Processes in an Electron-Ionization CO laser [in Russian], Preprint No. 183, FIAN, Moscow (1984).

PLASMA DYNAMICS OF OPTICAL BREAKDOWN DURING DEEP MELTING OF METALS

R. V. Arutyunyan, L. A. Bol'shov,
M. F. Kanevskii, K. A. Krivoruchko,
V. P. Reshetin, and R. I. Soloukhin*

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The perspectives for using pulsed-periodic CO_2-lasers for dimensional machining (laser cutting, welding, and drilling) has stimulated experimental and theoretical studies for the reaction of pulsed-periodic (PP) radiation with metals. A thermohydrodynamic (TH) model was developed in [1-4] for deep melting by pulsed-periodic radiation. According to [1-4] movement of melt in the cavity is cyclic. Retention of the melt on walls occurs as a result of vertical melt acceleration during the vapor pressure pulse and subsequent retardation of its movement by gravitation and capillary forces. The average temperature in the optimum regime for reaction is close to the melting temperature $T \sim T_m$. Equations for thermal balance within the model of a linear heat source and melt movement [1-4] make it possible to determine the optimum energy E and pulse frequency f, and to study their dependence on depth h, cavity radius a, beam displacement velocity v and target thermophysical properties, shape and duration of the radiation pulse. In particular, with prescribed cavity parameters a and h and velocity v the optimum laser pulse (LP) energy is only governed by its duration.

Within the limits of the TH-model for deep melting by PP-radiation it was assumed a priori that the required energy was put into the cavity and its absorption over the depth was uniform. The question remains open about whether radiation parameters are sufficient for melting large thicknesses of metal taking account of radiation propagation through the plasma above the metal surface and within the deep melting channel.

The plasma of optical breakdown above the metal surface may transform considerably the space-time structure of the laser pulse up to complete screening of the cavity. Currently the question of passage of a LP through a jet with different duration of pulses and the provisional shape of them has not been studied sufficiently either theoretically or experimentally. However, the effect of the jet above the metal surface with deep melting may be avoided by reducing the pressure and selecting the surrounding gas and the provisional shape of the pulse. For example, for radiation pulses with a leading spike with an intensity exceeding the threshold for a light-detonation wave (LDW) a regime appears to be possible when the "tail" of radiation pulses passes through the LDW plasma channel and when its expansion leads to almost complete restoration of transparency. Limitations connected with the radiation propagation processes within the deep melting cavity and optical breakdown in a vapor-gas mixture in the channel are fundamental in character.

Numerical calculation of CO_2-radiation propagation by the procedure in [5] in metal channels with a shape close to that observed experimentally in welding with considerable tapering (~20) indicate that LP self-absorption at the walls of the channel in the absence of optical breakdown is small (at the level of 10%) and it does not provide uniform insertion of a significant part of the energy.

One of the mechanisms for putting LP energy in effectively may be release of absorbed energy by the plasma within the deep melting cavity. With this approach limitations develop on the pulse duration τ_u, and from the selection of optimum curves for energy and pulse repetition frequency corresponding to different τ_u, one is separated.

*Deceased.

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